**Graphic Equalizer Design Using Higher-Order Recursive Filters**

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**Graphic Equalizers**

- control gain in a number of adjacent bands
- sliders' knobs represent samples of the frequency response
- band center frequencies distributed logarithmically:
  \[ f_{c,i} = R_i \cdot f_{c,i-1} \]
  \[ R = \frac{2}{\sqrt{3}} \] for octave equalizer
  \[ R = \frac{2}{\sqrt{5}} \] for 1/3-octave equalizer
- band edges: geometric mean of center frequencies
  \[ f_{e,i} = f_{c,i}/\sqrt{R_i} \]
  \[ f_{e,i} = f_{c,i} \cdot \sqrt{R_i} = f_{c,i-1} \]
- bandpasswidth:
  \[ \Delta f_i = f_{e,i} - f_{c,i} = f_{c,i} \left( \sqrt{R_i} - \frac{1}{\sqrt{R_i}} \right) \]

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**Band Shelving Filters**

**Desired properties**

- easy specification of the gain in the band
  \[ |H(e^{j\omega})| = g \]
- near unity gain outside the band
  \[ |H(e^{j\omega})| \approx 1, \omega \notin \Omega_1, \Omega_2 \]
- complementary filter edges, especially half the gain in dB at the edge frequencies
  \[ |H(e^{j\omega_0})| = |H(e^{j\omega_{1/2}})| = \sqrt{g} \]

**Example octave equalizer**

Order of band-shelving filters: \( B = M - 1 \) (\( M = 4 \))

Different gain in adjacent bands

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 Hz</td>
<td>6 dB</td>
</tr>
<tr>
<td>1 kHz</td>
<td>12 dB</td>
</tr>
<tr>
<td>10 kHz</td>
<td>0 dB</td>
</tr>
</tbody>
</table>

- no significant influence across bands

Same gain in all bands

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 Hz</td>
<td>0 dB</td>
</tr>
<tr>
<td>1 kHz</td>
<td>0 dB</td>
</tr>
<tr>
<td>10 kHz</td>
<td>0 dB</td>
</tr>
</tbody>
</table>

- nearly constant amplitude, except for transitional region below highest band

Reduce \( f_{c,i} \) of highest band from 21.7 kHz to 18.5 kHz

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 Hz</td>
<td>14 dB</td>
</tr>
<tr>
<td>1 kHz</td>
<td>14 dB</td>
</tr>
<tr>
<td>10 kHz</td>
<td>14 dB</td>
</tr>
</tbody>
</table>

- nearly constant amplitude across whole frequency axis

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**Implementation**

Realization of the shelving filters as a cascade fourth-order sections

For each band:

\[ K = \frac{1}{\sqrt{R_i}} \tan \left( \frac{\Omega_{b,i}}{2} \right) \]
\[ V = \sqrt{g} \]
\[ A(z) = z^{-1} \cos(\Omega_{b,i} z^{-1}) \]

For each section \( m = 1, \ldots, B \) in each band:

\[ \cos(\Omega_{b,i} z^{-1}) = \cos(A_{2n,m}) = \frac{1}{2} \left[ 1 + 2K_{2n,m} z^{-1} \right] \]

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**Java demo applet**

http://ant.hsu-hh.de/jdafx