Robust Design of Very High-Order Allpass Dispersion Filters

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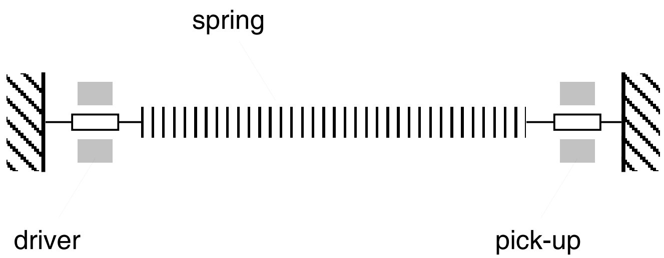
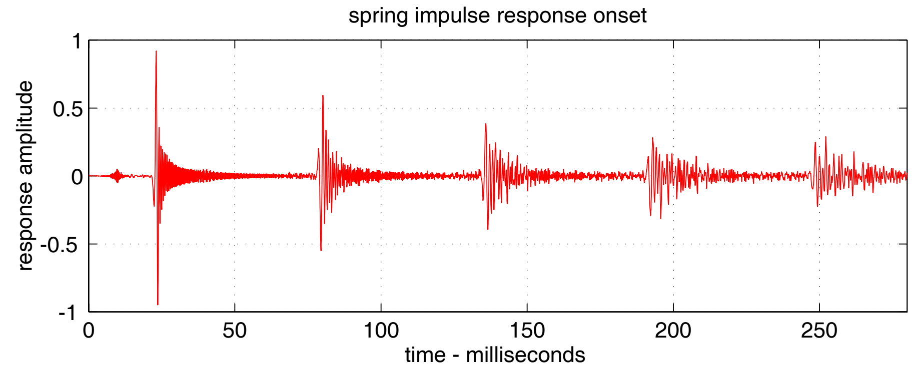
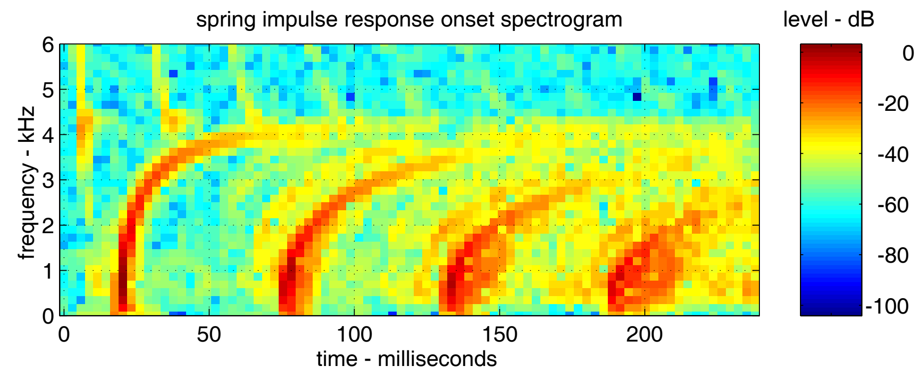
Dispersive Propagation in a Spring Reverb



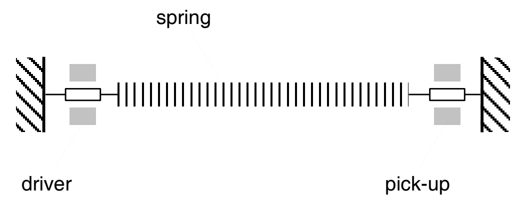
Accutronics Type 8 Spring Reverberator

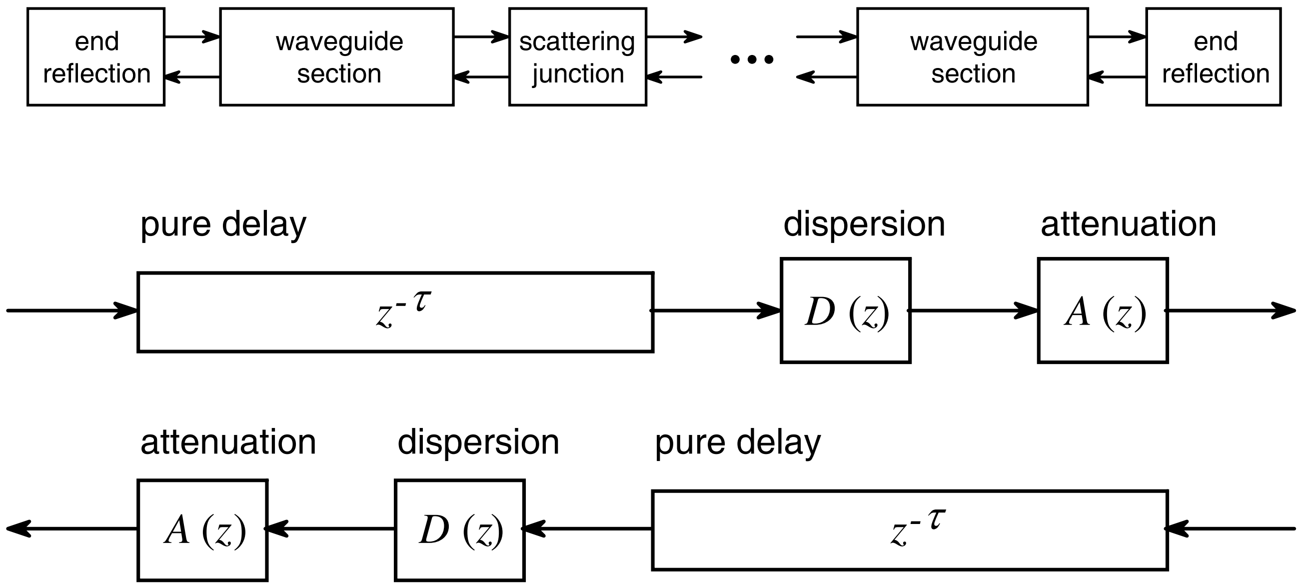




- **The torsional mode typically used by spring reverberators is highly dispersive, giving the spring its characteristic sound.**

Dispersive Waveguide





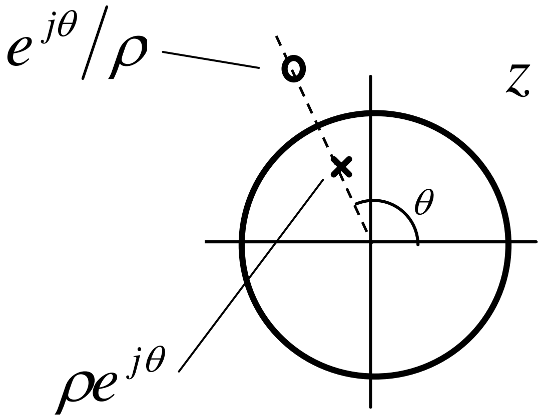
- **Each spring element is modeled using a set of dispersive waveguide sections.**
- **Left-going and right-going waves are separately processed via delay elements and commuted dispersion and loss filters.**

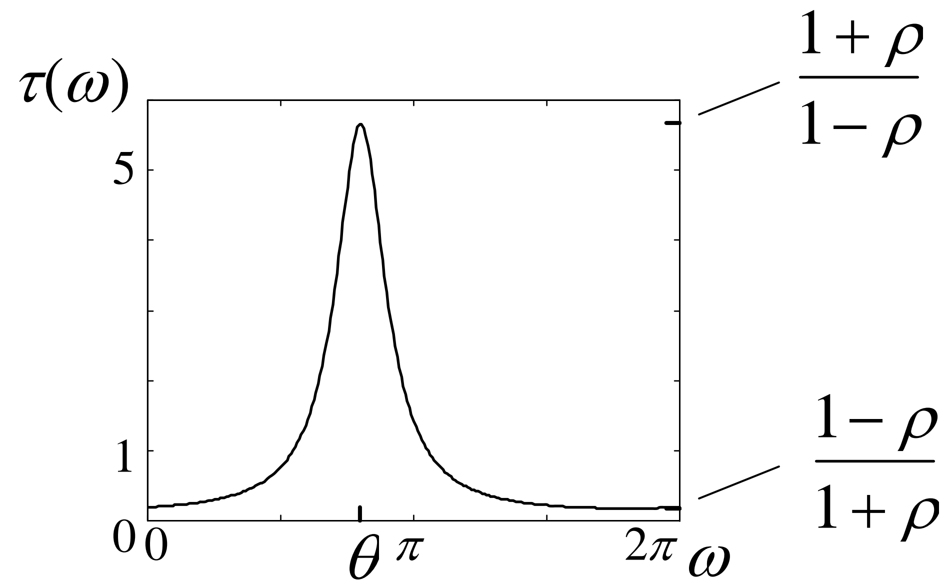
Allpass Dispersion Filter Design

$$G(z) = \frac{\rho_N + \rho_{N-1} z^{-1} + \cdots + \rho_1 z^{-N+1} + z^{-N}}{1 + \rho_1 z^{-1} + \cdots + \rho_{N-1} z^{-N+1} + \rho_N z^{-N}}$$

- **Hilbert transform methods**
  - **Yegnanarayana, IEEE-ASSP 1982**
  - **Reddy and Swamy, ICASSP 1998**
  - **Filter phase (not group delay) matched**
  - **Potential time aliasing, numerical issues; not in factored form**
- **Optimal filter design formulation**
  - **Lang and Laakso, IEEE-CAS1 1994; Lang, IEEE-SP 1998**
  - **Rocchesso and Scalcon, IEEE-CAS 1996**
  - **Bensa et al., ASA 2004**
  - **Rauhala and Valamaki, IEEE-SPL 2006**
  - **Maximum order limited by numerical difficulties; expensive design**

First-Order Allpass Filter





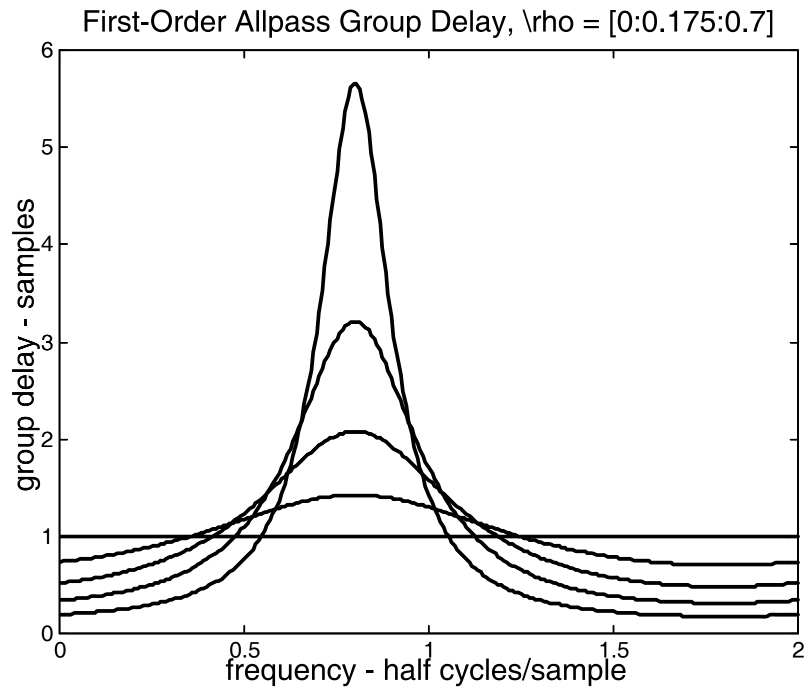
$$G(z) = \frac{-\rho e^{-j\theta} + z^{-1}}{1 - \rho e^{j\theta} \cdot z^{-1}}$$

$$\tau(\omega) = \frac{1 - \rho^2}{1 + \rho^2 - 2\rho\cos(\omega - \theta)}$$

transfer function

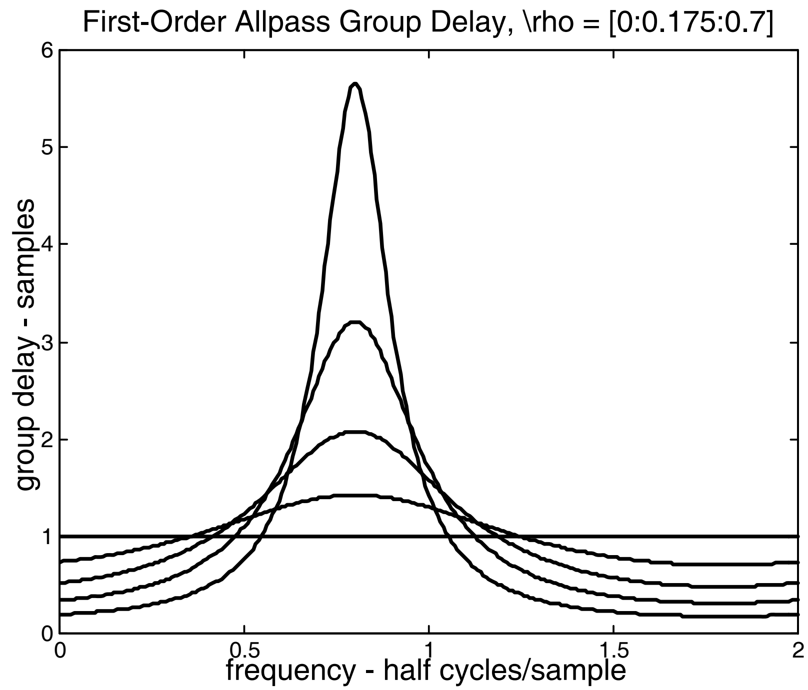
group delay

First-Order Allpass Group Delay



- **As the pole is moved toward the unit circle, the first-order allpass group delay $\tau(\omega)$ becomes more peaked about the pole angle θ – the maximum increases, and the peak narrows.**

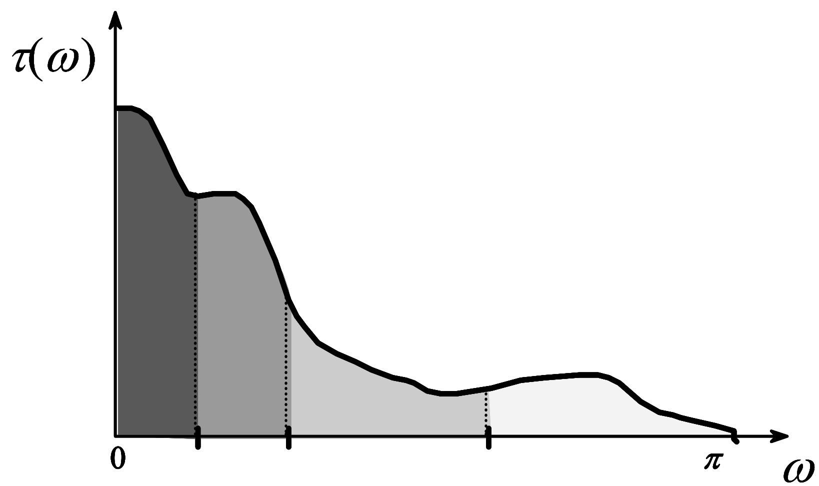
First-Order Allpass Group Delay



- **The integral of the group delay $\tau(\omega)$ of a first-order allpass filter is 2π, independent of ρ and θ,**

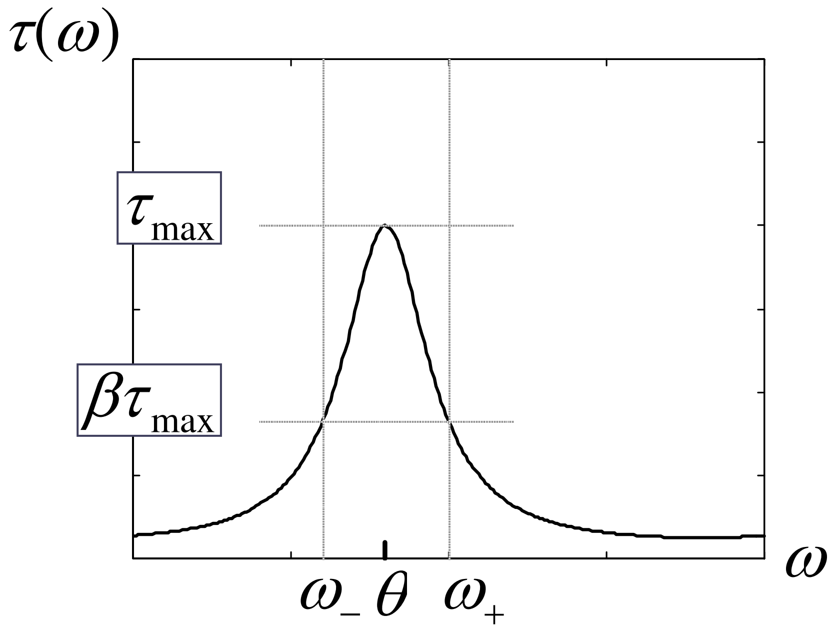
$$\int_0^{2\pi} \tau(\omega)d\omega = \varphi(2\pi) - \varphi(0) = 2\pi.$$

Allpass Filter Design Approach



- **Integrate** $\tau(\omega)$**, and add a constant delay** τ_0 **such that** $\tau(\omega) + \tau_0$ **integrates to a multiple of** 2π**.**
- **Divide** $\tau(\omega) + \tau_0$ **into** 2π **-area frequency bands.**
- **Fit a first-order allpass filter section to each band.**

First-Order Allpass Design



$$\rho = \eta - \left[\eta^2 - 1\right]^{1/2}$$

$$\eta = \frac{1 - \beta\cos\delta}{1 - \beta}$$

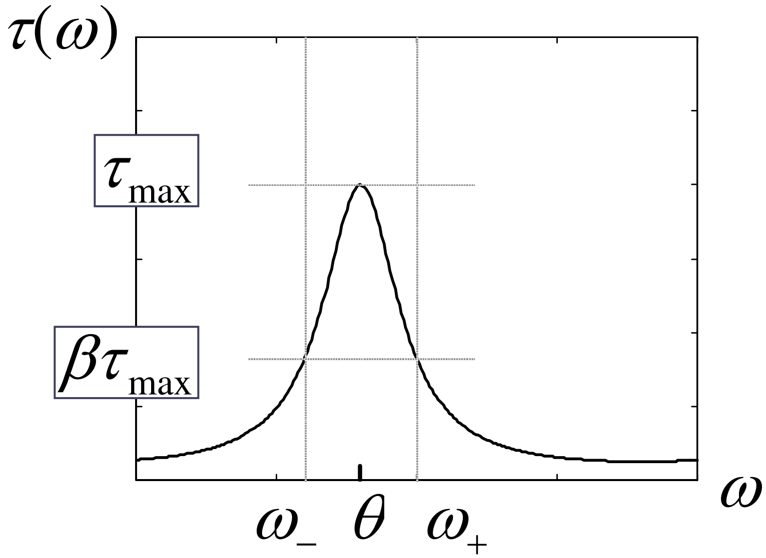
$$\delta = (\omega_- - \omega_+)/2$$

- **The pole angle θ is the band midpoint,**

$$\theta = (\omega_- + \omega_+)/2$$

- **The section pole radius ρ is chosen to make the band edge group delay a fraction β of its maximum.**

Dispersion Filter Design Cost

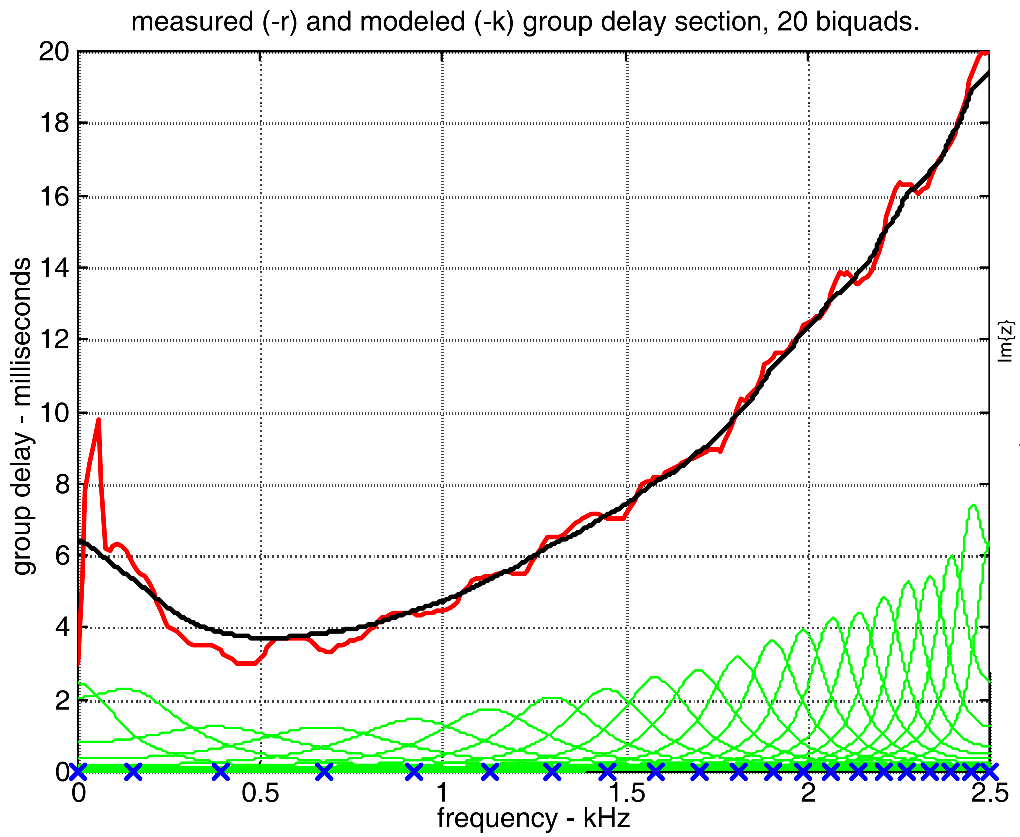


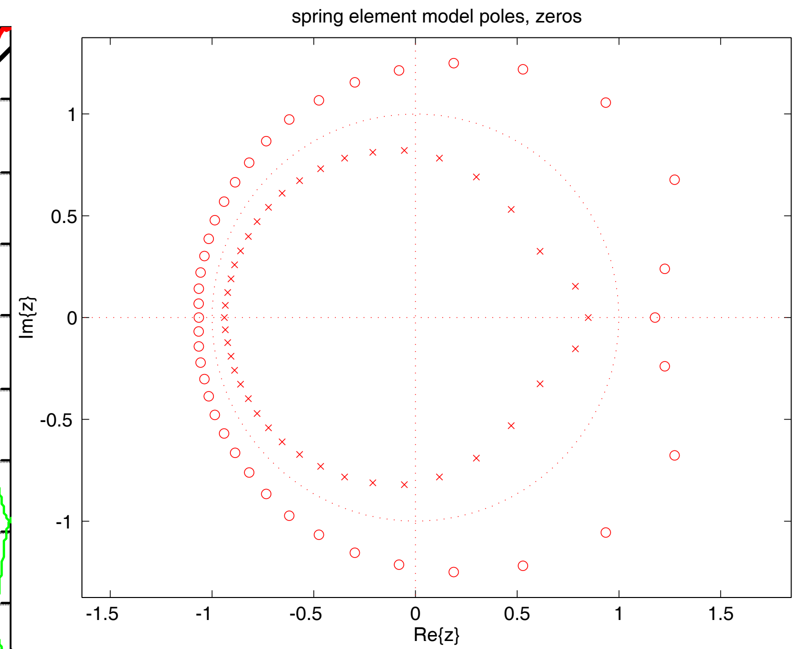
$$\rho \approx 1 - \left[\frac{\beta}{1-\beta}\right]^{1/2}\delta, \;\; \delta << 1$$

$$\delta = \tfrac{1}{2}\left|\omega_+ - \omega_-\right|$$

- **The design method is very inexpensive and may be used to update dispersion filters in real time.**
  - **The pole angles θ_k directly encode the dispersive delay $\tau(\omega)$, and may be efficiently computed.**
  - **The pole radii $\rho_k(\beta)$ control delay smoothing, and are roughly linear in section bandwidth.**

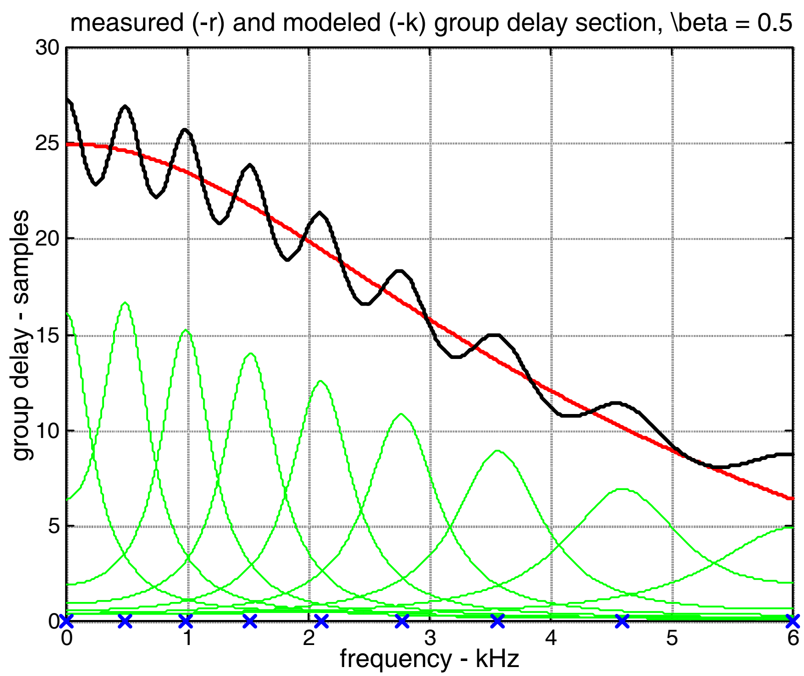
Design Example: Spring Reverberator Element

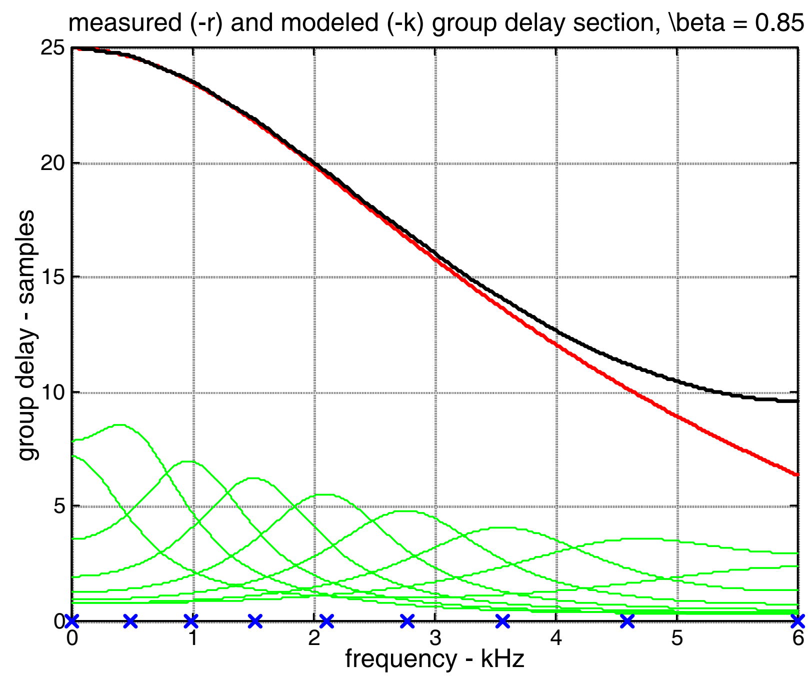




- **Poles, zeros follow smooth trajectories.**

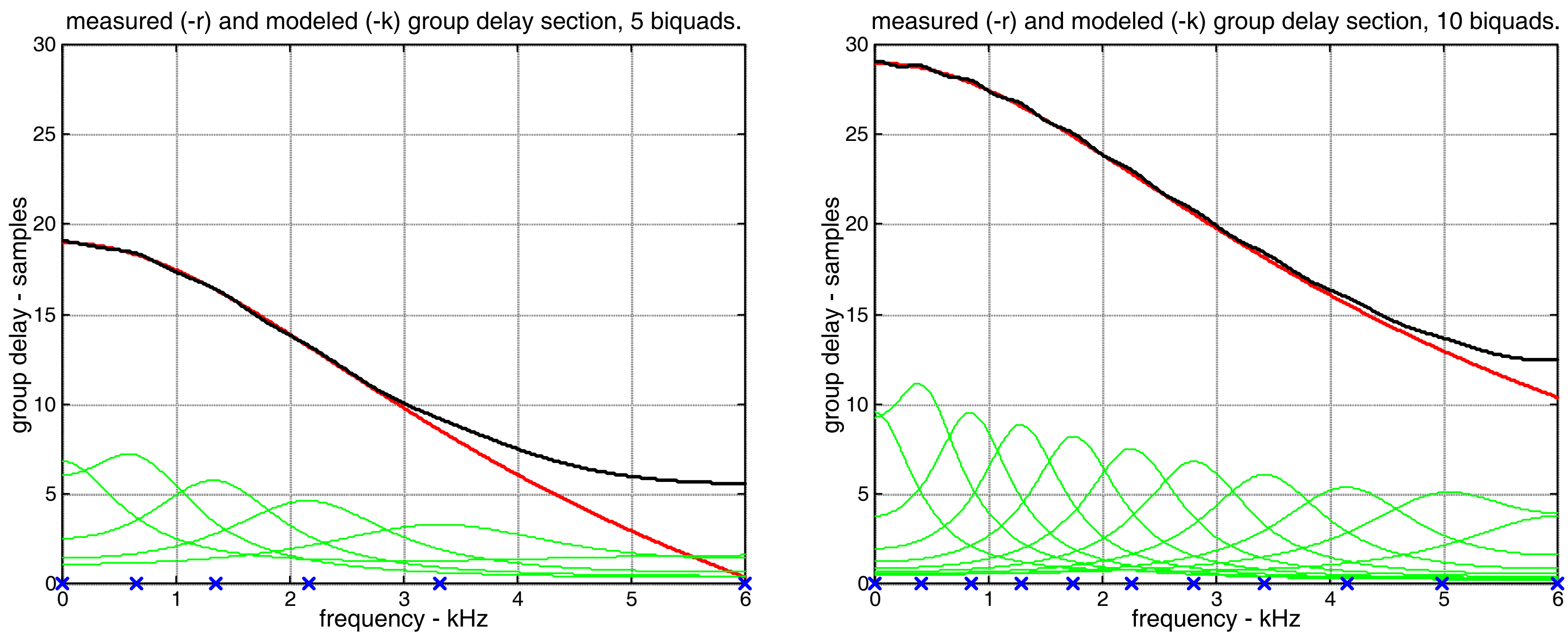
Adjusting β





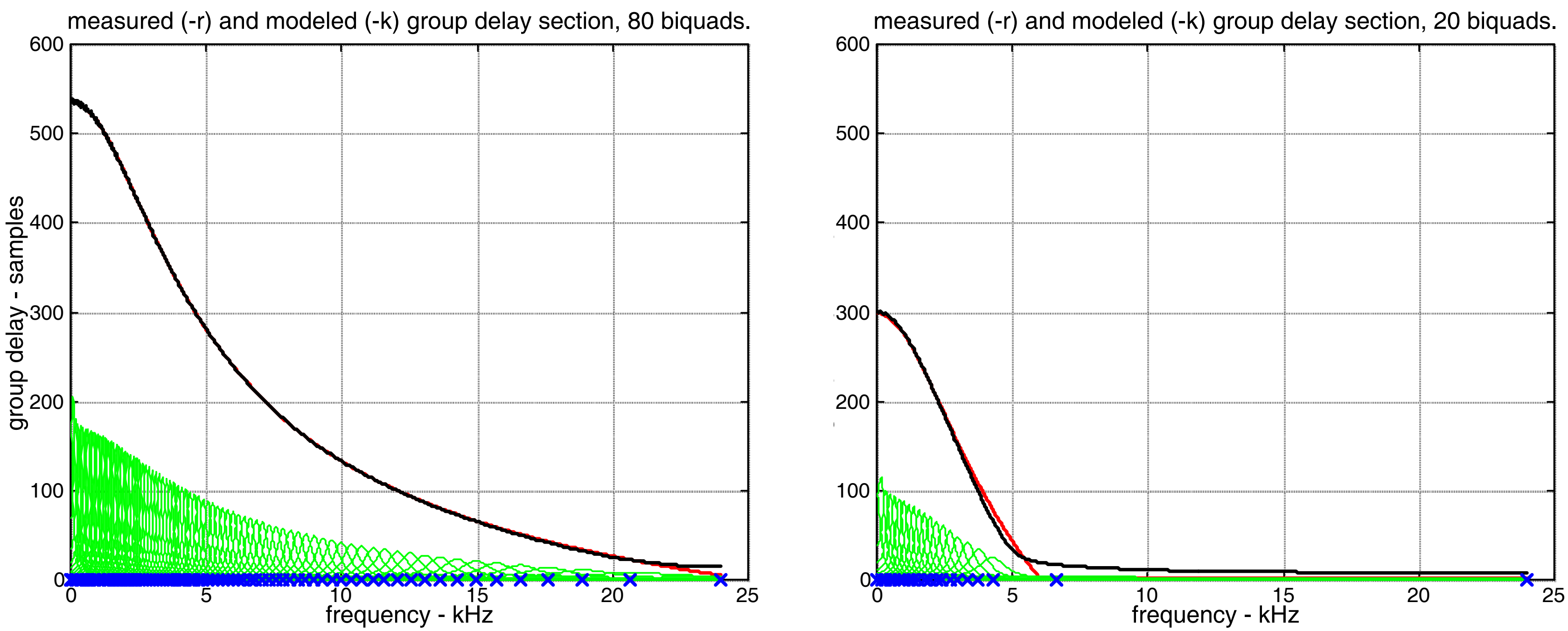
- **Adjusting β trades ripple for responsiveness to narrow-band group delay changes.**

Increasing Model Order



- **Adding a constant delay τ_0 to the group delay $\tau(\omega)$ allows additional allpass sections to be used, and provides a more accurate fit.**

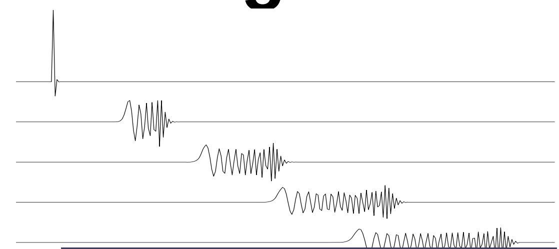
Low-Frequency Modeling



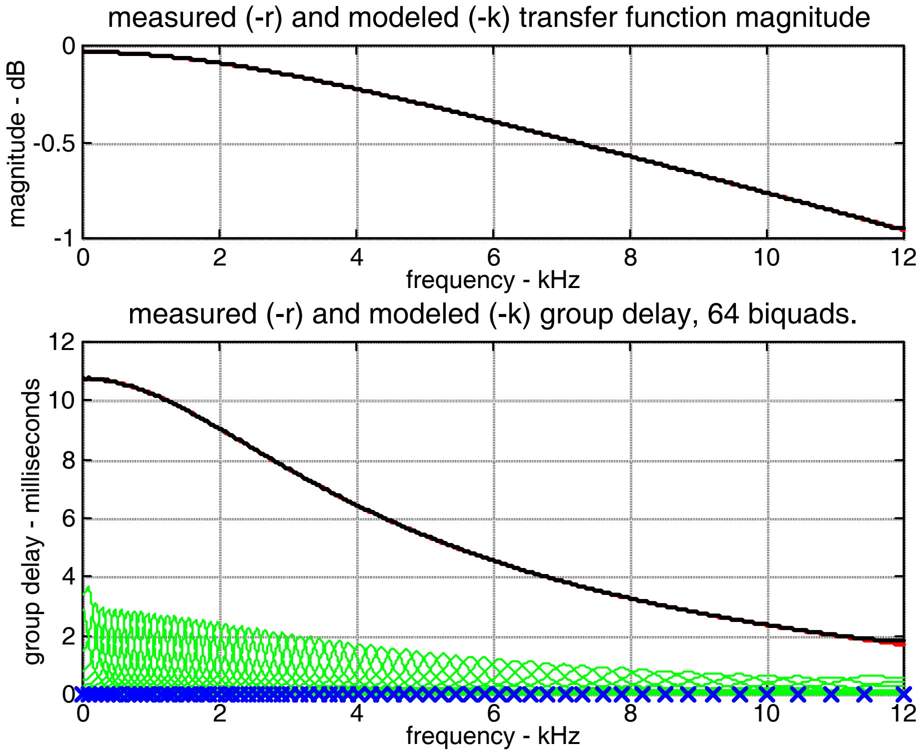
- **By setting $\tau(\omega) = 0$ outside the band of interest, the model order may be reduced.**

Piano String Propagation Filter Design

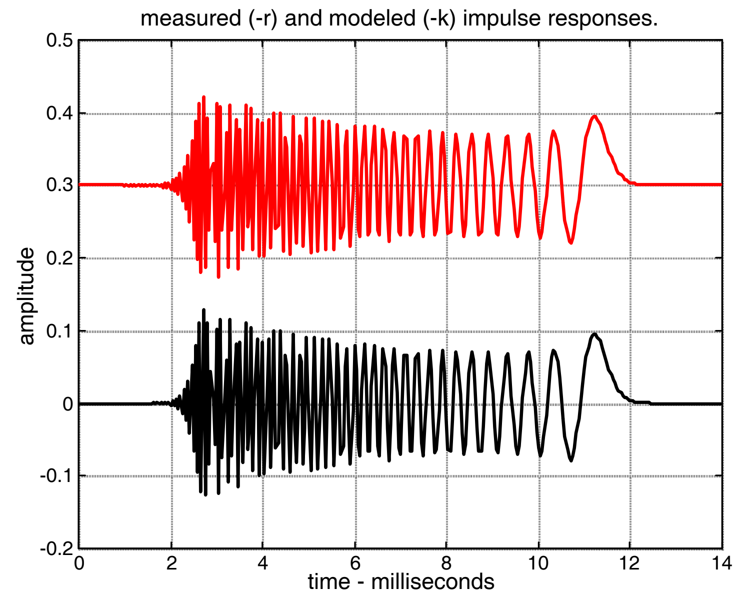
$$\exp\{-\alpha(\omega)\cdot d - j[\omega / c_0 - \varphi(\omega)]\cdot d\}$$



string displacement



transfer functions

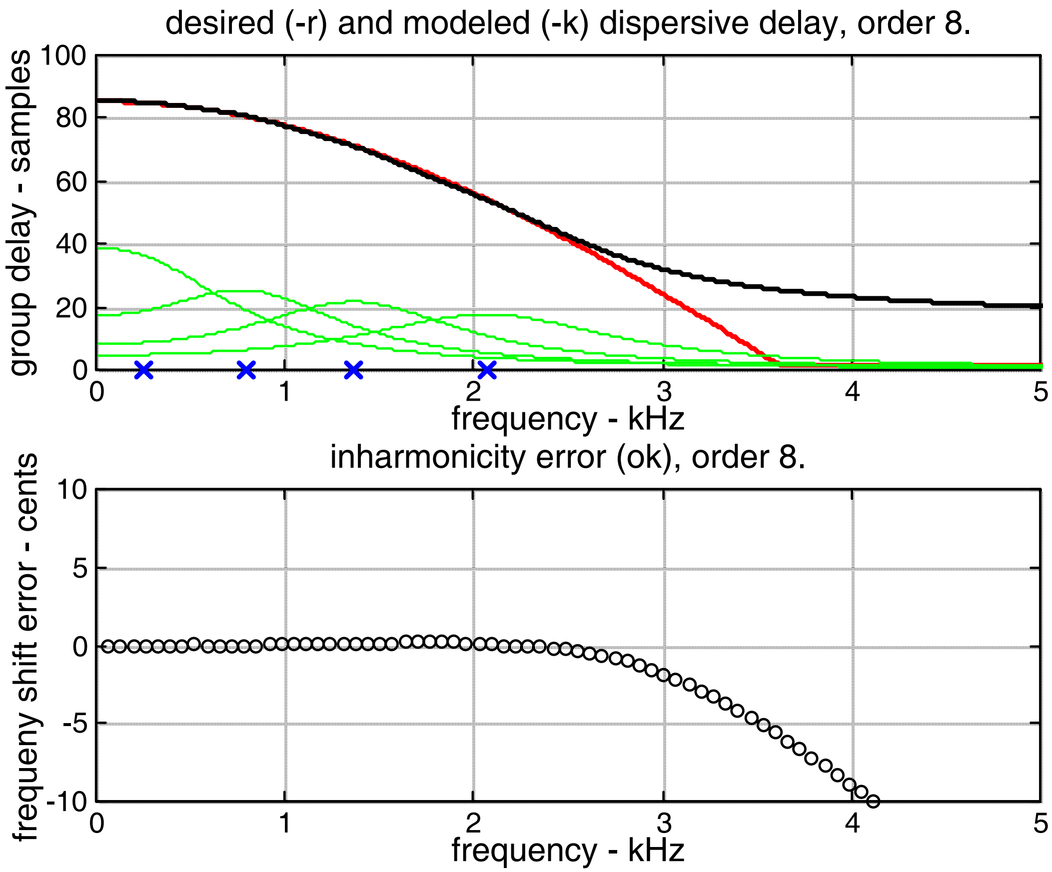


impulse responses

Stiff String Propagation Filter Design

$$\tau(\omega) = \frac{\tau_0}{\sqrt{1 + B\omega^2}}$$

$$\varphi(\omega) = \frac{\tau_0}{\sqrt{B}} \cdot \operatorname{asinh}\sqrt{B}\omega$$



Summary

- **New method for allpass dispersion-filter design:**
  - **Simple, numerically robust, nonparametric**
  - **Model order automatically determined**
  - **Filters produced in factored biquad form**
- **Future work**
  - **Applications**
    - Strings, springs and tubes of all kinds
    - Filter group-delay equalization
  - **Extensions**
    - Multiband group-delay filter design
    - Time-varying group-delay design
    - Frequency-dependent smoothing parameter β